MORE PRACTICAL CRITICAL HEIGHT SAMPLING

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Critical Height Sampling (CHS) (Kitamura 1964) can be used to predict cubic volumes per acre without using volume tables or equations. The critical height is defined as the height at which the tree stem appears to be in borderline condition using the point-sampling angle gauge (e.g. prism). An estimate of cubic volume per acre can be obtained from multiplication of the sum of the critical heights at a sample point by the point sampling basal area factor. One of the most serious problems with practical implementation of critical height sampling is that trees near the sample point have a very high critical height, which can be difficult to view from the sample point. It is proposed to correct this by obtaining the "antithetic variate" associated with each tree which is:

$$(1-u^*) = (1 - b/B) \tag{1}$$

where b is the cross-sectional area at critical height of the stem (equal to stem cross-sectional area at borderline) and B is the basal area of the tree.

The value of b can be computed from the distance to the tree, while the value of B will be based on measurements of diameter at breast height (d.b.h.). This 1-u* is used to perform importance sampling on sample trees. This will result in measurement of height to an upper-stem diameter on each sample tree, which will be lower on trees near the sample point and elevated as trees are more distant from the sample point.

Importance sampling is a method of obtaining unbiased tree-volume estimates using randomly selected upper-stem tree heights or diameters. Under importance sampling, a proxy taper function which approximates actual tree shape is used to sample tree dimensions with probability density proportional to proxy volume. We used an importance sampling individual tree volume estimator developed by Lynch and others (1992) using a paraboloid as a proxy taper function. Importance sampling can be combined with critical height sampling by using the following uniform random variate:

$$u^* = b/B \tag{2}$$

where b is the cross-sectional area at critical height for tree i and also the "borderline" cross-sectional area for a tree located at the same distance from the sample point as tree i.

This uniform random variate was then used in developing an estimator for volume per unit area which uses the importance sampling individual tree volume estimate in place of actual individual tree volume in the classic Horizontal Point Sampling (HPS) estimator. We refer to this estimator as the Importance sampling Critical Height Sampling (ICHS) estimator.

With ICHS, the upper-stem height measurement, h(b), is located high on the stem for trees close to the sample point and low on the stem for trees distant from the sample point. However, we can reverse that trend by using the antithetic variate, 1-u*, in Lynch and others (1992) importance sampling individual tree volume estimator. We refer to this estimator as the Antithetic Importance sampling Critical Height Sampling (AICHS) estimator. This solves the problem of steep viewing angles for CHS sample trees near the sample point because the relationship between distance-to-tree and upper-stem viewing height is reversed when 1-u* is used instead of u.

The sampling surface simulator of Gove (2012) was adapted and used to perform simulations to compare the precisions of CHS, ICHS, and AICHS to ordinary HPS. HPS is the "gold standard" because individual tree volumes in HPS are assumed known without error.

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Sampling surfaces are generated by evaluating each estimator at every point on a fine grid system covering the simulation tract. Sampling simulations with metric F = 4 were conducted on a reconstruction of a mature shortleaf pine (*Pinus echinata* Mill.) forest having 90 square feet of basal area per acre at age 80 and site index 50 feet at age 50, based on data for mature shortleaf from Huebschmann (2000).

HPS, CHS, AICHS, and ICHS are known to be unbiased from theoretical results since, for each of these methods, the mathematical expected value of the estimator is equal to forest volume for the sampled tract. However it should be noted that the lack of bias in HPS depends on the assumption that individual sample tree volumes come from an unbiased volume table or equation. CHS, AICHS, and ICHS do not require this assumption since they estimate tree volumes based on upper-stem stem measurements in the field. Simulation results empirically confirmed the lack of bias of each estimator. Simulation results indicated that ICHS, AICHS, and HPS had lower standard deviations and therefore were more precise than CHS. Very notable is that the new methods, ICHS and AICHS, are equally as precise as HPS in which individual tree volumes are known without error. This is very important because the new methods, like critical height sampling, avoid bias inherent in volume equations or tables to which HPS is subject.

LITERATURE CITED

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